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Bottom-Up Pyramid Cellular Acceptors with Three-Dimensional Layers

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Abstract

In 1997, C.R.Dyer and A.Rosenfeld introduced an acceptor on a two-dimensional pattern (or tape), called the *pyramid cellular acceptor*, and demonstrated that many useful recognition tasks are executed by pyramid cellular acceptors in time proportional to the logarithm of the diameter of the input. They also introduced a *bottom-up pyramid cellular acceptor* which is a restricted version of the pyramid cellular acceptor, and proposed some interesting open problems about bottom-up pyramid cellular acceptors. On the other hand, we think that the study of three-dimensional automata has been meaningful as the computational model of three-dimensional infomation processing such as computer vision, robotics, and so forth. In this paper,we investigate about bottom-up pyramid cellular accptors with three-dimensional layers, and show their some accepting powers.

Key Words **:** cellular automaton, diameter, finite automaton, pattern recognition, three-dimension.

1 Introduction

M.Blum and C.Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing, and investigated their pattern recognition abilities [1]. Since then, many researchers in this field have been investigating a lot of properties about automata on a two-dimensional tape. In [2], C.R.Dyer and A.Rosenfeld introduced an acceptor on a two-dimensional pattern (or tape), called the *pyramid cellular acceptor*, and demonstrated that many useful recognition tasks are executed by the pyramid cellular acceptors in time proportional to logarithm of the diameter of the input. They also introduced a *bottom-up pyramid cellular acceptor*, which is a restricted version of the pyramid cellular acceptor, and proposed some interesting open problems about it. On the other hand, the question of whether processing three-dimensional digital patterns is much difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the study of three-dimensional automata as the computasional model of three-dimensional pattern processing has been meaningful. From this point of view, we are interested in three-dimensional automata.

In this paper, we study about bottom-up pyramid cellular acceptors with three-dimensional layers, and deal with the following problems (which is one of the open problems) : Does the class of sets accepted by deterministic bottom-up pyramid cellular acceptors with three-dimensional layers include the class of sets accepted by deterministic *three-dimensional finite automata* [3-7]? This paper shows that the class of sets accepted by three-dimensional finite automata is incomparable with the class of sets accepted by deterministic bottom-up pyramid cellular acceptors which operate in time of order lower than the *diameter* of the input.

2 Definition

Let Σ be a finite set of symbols. A *three-dimensional tape* over Σ is a three-dimensional rectangular array of elements of Σ. The set of all the three-dimensional tapes over Σ is denoted by $\Sigma^{(3)}$. Given a tape $x \in \Sigma^{(3)}$, for each j $(1 \leq j \leq 3)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, and $l_3(x) = n_3$ is denoted by $\Sigma^{(n_1\ n_2\ n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each j $(1 \leq j \leq 3)$, let x (i_1, i_2, i_3) denote the symbol in x with coordinates (i_1, i_2, i_3). Furthermore, we define x $[(i_1, i_2, i_3), (i_1', i_2', i_3')]$, when $i \leq i_j \leq i_j' \leq l_j(x)$ for each integer j $(1 \leq j \leq 3)$, as the three-dimensional input tape y satisfying the following (i) and (ii) : (i) for each j $(1 \leq j \leq 3)$, $l_j(y) = i_j' - i_j + 1$; (ii) for each r_1, r_2, r_3 $(1 \leq r_1 \leq l_1(y), 1 \leq r_2 \leq l_2(y), 1 \leq r_3 \leq l_3(y))$, y $(r_1, r_2, r_3) = x$ $(r_1 + i_1 - 1, r_2 + i_2 - 1, r_3 + i_3 - 1)$.

For each $x \in \Sigma^{(n_1\ n_2\ n_3)}$ and for each $1 \leq i_1 \leq n_1$, $1 \leq i_2 \leq n_2$, $1 \leq i_3 \leq n_3$, $x[(i_1, 1, 1), (i_1, n_2, n_3)]$, $x[(1, i_2, 1), (n_1, i_2, n_3)]$, $x[(1, 1, i_3), (n_1, n_2, i_3)]$, $x[(i_1, 1, i_3), (i_1, n_2, i_3)]$, and $x[(1, i_2, i_3), (n_1, i_2, i_3)]$ are called the i_1th (2-3) *plane* of x, the i_2th (1-3) *plane* of x, the i_3th (1-2) *plane* of x, the i_1th *row* on the i_3th (1-2) *plane* of x, and the i_2th *column* on the i_3th (1-2) *plane* of x.

We next give some basic concepts about bottom-up pyramid cellular acceptors with three-dimensional layers [7]. A *bottom-up pyramid cellular acceptor with* *three-dimensional layers* (3-$UPCA$) is a pyramidal stack of three-dimensional arrays of cells in which the bottom three-dimensional layer has size $2^n \times 2^n \times 2^n$ ($n \geq 0$), the next lowest $2^{n-1} \times 2^{n-1} \times 2^{n-1}$, and so forth, the $(n+1)$st three-dimensional layer consisting of a single cell, called the *root*. Each cell is defined as an identical finite-state machine, $M = (Q_N, Q_T, \delta, A)$, where Q_N is a nonempty, finite set of states, $Q_T \subseteq Q_N$ is a finite set of *input states*, $A \subseteq Q_N$ is the set of *accepting states*, and $\delta : Q_N^9 \rightarrow Q_N$ is the *state transition function*, mapping the current states of M and its eight son cells in a $2 \times 2 \times 2$ block on the three-dimensional layer below into M's next state. As shown in Fig.1, let c be some cell on the $(i+1)$st three-dimensional layer, and let c (UNW), c (USW), c (USE), c (UNE), c (DNW), c (DSW), c (DSE), and $c(DNE)$ be eight son cells (on the ith three-dimensional layer) of c, where $c(UNW)$ is c's upper northwest son, c (DNW) is c's lower northwest son, etc. For example, if the coordinates of c on the $(i+1)$st layer is (1, 1, 1) $((2^n, 2^n, 2^n))$, the coordinates of eight son cells of c on the ith layer $c(UNW)$, $c(USW)$, $c(USE)$, c(UNE),c(DNW),c(DSW),$c(DSE)$, and $c(DNE)$ are (1, 1, 1), (2, 1, 1), (2, 2, 1), (1, 2, 1), (1, 1, 2), (2, 1, 2), (2, 2, 2), (1, 2, 2), $((2^n-1, 2^n-1, 2^n-1)$, $(2^n, 2^n-1, 2^n-1)$, $(2^n, 2^n, 2^n-1)$, $(2^n-1, 2^n, 2^n-1)$, $(2^n-1, 2^n-1, 2^n)$, $(2^n, 2^n-1, 2^n)$, $(2^n, 2^n, 2^n)$, $(2^n-1, 2^n, 2^n))$, respectively. Then $q_c(t+1) = \delta$ $(q_c(t), q_{c(UNW)}(t), q_{c(USW)}(t), q_{c(USE)}(t), q_{c(UNE)}(t), q_{c(DNW)}(t), q_{c(DSW)}(t), q_{c(DSE)}(t), q_{c(DNE)}(t))$, where for example $q_c(t)$ means the state of c at time t. At time $t = 0$, the input tape $x \in Q_T^{(3)}$ $[l_1(x) = l_2(x) = l_3(x) = 2^n, n \geq 0]$ is stored as the initial states of the bottom three-dimensional layer, henceforth called the *base*, in such a way that x (i_1, i_2, i_3) is stored at the cell of the i_1th row and the i_2th column on the i_3th plane, and the other cells are initialized to a *quiescent state* q_s $(\in Q_N - Q_T - A)$. As usual, we let δ $(q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s) = q_s$. The input is *accepted* if and only if the root cell ever enters an accepting state. This 3-$UPCA$ is called *deterministic*. A *nondeterministic bottom-up pyramid cellular acceptor* is defined as a 3-$UPCA$ using $\delta : Q_N^9 \rightarrow 2^{Q_N}$ instead of the state transition function of the deterministic 3-$UPCA$. Below, we denote a deterministic 3-$UPCA$ by 3-$DUPCA$, and a nondeterministic 3-$UPCA$ by 3-$NUPCA$. A 3-$DUPCA$ (or 3-$NUPCA$) operates in time $T(n)$ if for every three-dimensional tape of size $2^n \times 2^n \times 2^n$ ($n \geq 0$) it accepts the three-dimensional tape, then there is an accepting computation which uses no more than time $T(n)$. By 3-$DUPCA$ $(T(n))$ [3-$NUPCA(T(n))$] we denote a $T(n)$ time-bounded 3-$DUPCA$ [3-$NUPCA$] which operates in time $T(n)$.

We next recall a three-dimensional finite automaton [8]. A *three-dimensional finite automaton* (3-FA) is a three-dimensional Turing machine with no workspace. A 3-FA M has a read-only three-dimensional tape with boundary symbols #'s, finite control, and an input head, as shown in Fig.2. The input head can move in six direction − east, west, south, north, up, or down − unless it falls off the input tape. Formally, M is defined by the 5-tuple $M = (K, \Sigma \cup \{\#\}, \delta, q_0, F)$, where K is a finite set of *states*, Σ is a finite set of *input symbols*, # is the *boundary symbol* (not in Σ), $\delta : K \times (\Sigma \cup \{\#\}) \rightarrow 2^{K \times \{E\ W\ S\ N\ U\ D\ H\}}$ is the *state transition function*, where E, W, S, N, U, D, and H represent the move directions of the input head − east, west, south, north, up, down, and no move, respectively, $q_0 \in K$ is the *intial state*, and $F \subseteq K$ is the set of *accepting states*. The action of M is similar to that of the one-dimensional (or two-dimensional) finite automaton [4], except that the input head of M can move in six directions. That is, when an input tape $x \in \Sigma^{(3)}$ with boundary symbols is presented to M, M starts in its initial state q_0 with the input head on x (1, 1, 1), and determines the next state of the finite control and the move direction of the input head, depending on the present state of the finite control and the symbol read by the input head. We say that M *accepts* the tape x if it eventually enters an accepting state. We denote a deterministic 3-FA [nondeterministic 3-FA] by 3-DFA [3-NFA].

We let the input tapes, throughout this paper, be restricted to cubic ones. We denote the set of all three-dimensional tapes accepted by M by $T(M)$. Define £ [3-$DUPCA$] = [T | $T(M)$ is accepted by some 3-$DUPCA$ M}. £ [3-$NUPCA$], £ [3-DFA], etc. are

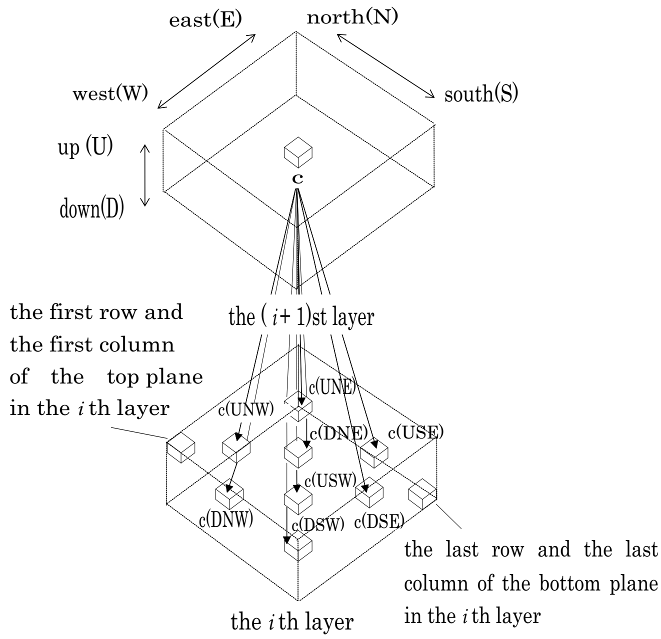

Fig. 1: Bottom-up pyramid cellular acceptor and three-dimensional layer.

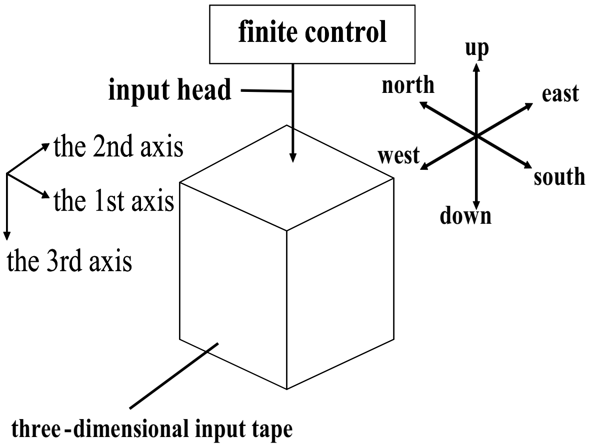

Fig. 2: Three-dimensional finite automaton.

defined similarly.

Finally, we give definition of *diameter*. Given a subset S of a tape $x \in \Sigma^{(3)}$, we can define its *extent* in a given direction θ as the length of its projection on a plane in that direction. Here the length of a projection is the distance between its farthest apart nonzero values. Thus the extent of S is the distance between a pair of parallel planes perpendicular to θ that just bracket S. The *diameter* of S is defined as its extent in any direction.

3 Results

In this section, we show that the class of sets accepted by *3-DFA's* is incomparable with the class of sets accepted by *3-DUPCA's* which operate in time of order lower than the diameter of the input. It has often been noticed that we can easily get several properties of three-dimensional automata by directly applying the results of one- or two-dimensional case, if the input tapes are not restricted to cubic ones. So we let the input tapes, throughout this paper, be restricted to cubic ones in order to increase the theoretical interest.

Lemma 3.1. *Let* $T_1 = \{ x \in \{ 0,1 \}^{(3)} \mid \exists n \; (n \geq 1) [\ell_1(x) = \ell_2(x) = \ell_3(x) = 2^n]$ *and* $x \; (2^{n-1}, 2^{n-1}, 2^{n-1}) = 1 \}$. *Then,*
(1) $T_1(x) \notin \pounds$ [3-*DFA*], *and*
(2) $T_1(x) \in \pounds$ [3-*DUPCA*(n)].

Proof : The Proof of (1) is similar to that of Theorem 3 in [7]. On the other hand, by using the same technique as in the proof of Lemma 1 in [6], we can get Part (2) of the lemma.

Lemma 3.2. *Let* $T_2 = \{x \in \{0,1\}^{(3)} \mid \exists n \; (n \geq 1) [\ell_1(x) = \ell_2(x) = \ell_3(x) = 2^n]$ *and* $x \; [(1,1,1), (2^n, 2^n, 1)] = x \; [(1,1,2^n), (2^n, 2^n, 2^n)] \}$. *Let* $T(n)$ *be a time function such that* $\lim_{n\to\infty} [T(n)/2^{2n}] = 0$. *Then,*
(1) $T_2 \in \pounds$[3-*DFA*],*and*
(2) $T_2 \notin \pounds$[3-*DUPCA*$(T(n))$].

Proof : It is obvious that there is a *3-DFA* accepting T_2, and so (1) of the lemma holds. Below, we prove (2). Suppose that there is a *3-DUPCA* B which accepts T_2 and operates in time $T(n)$, and that each cell of B has k states. For each $n \geq 2$, let
$W(n) = \{x \in \{0,1\}^{(3)} | \ell_1(x) = \ell_2(x) = \ell_3(x) = 2^n\}$, and

$$\begin{aligned} W'(n) \;\; = \;\; & \{x \in \{0,1\}^{(3)} | \ell_1(x) = \ell_2(x) = \ell_3(x) = 2^{n-1} \\ & \& \;\; x[(1,1,1),(2^{n-1},2^{n-1},1)] \in \{0,1\}^{(3)} \\ & \& \;\; x[(1,1,2),(2^{n-1},2^{n-1},2^{n-1})] \in \{0\}^{(3)}\}. \end{aligned}$$

We consider the cases when the tapes in $W(n)$ are presented to B. Let c be the cell which is situated at the first row, the first column, and the first plane in the nth layer (i.e., the layer just below the root cell). (Note that there are eight cells in the nth layer.) For each x in $W(n)$ such that $x[(1,1,1),(2^{n-1},2^{n-1},2^{n-1})] \in W'(n)$, and for each $r \geq 1$, let $q_r(x)$ be the state of c at time r when x is presented to B. Then the following proposition must hold.

Proposition 3.1. *Let* x, y *be two different tapes in* $W(n)$ *such that both* $x \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})]$ *and* $y \; [(1,1,1),\, (2^{n-1},2^{n-1},2^{n-1})]$ *are in* $W'(n)$ *and* $x \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})] \neq y \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})]$. *Then,* $(q_1(x), q_2(x), \ldots, q_{T(n)}(x)) = (q_1(y), q_2(y), \ldots, q_{T(n)}(y))$.

[**Proof :** For suppose that $(q_1(x), q_2(x), \ldots, q_{T(n)}(x)) = (q_1(y), q_2(y), \ldots, q_{T(n)}(y))$. We consider two tapes z, z' in $W(n)$ such that

(i) $z \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})]$
$\quad = x \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})]$ and
$z' \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})]$
$\quad = y \; [(1,\,1,\,1),\, (2^{n-1},2^{n-1},2^{n-1})]$,

(ii) the part of z except for $z \; [(1,\,1,\,1),\, (2^{n-1},\, 2^{n-1},\, 2^{n-1})]$ is identical with the part z' except for $z'[(1,1,1),(2^{n-1},2^{n-1},2^{n-1})]$,

and

(iii) $z[(1,1,1),(2^n,2^n,1)] = z[(1,1,2^n),(2^n,2^n,2^n)]$.

By assumption, the root cell of B enters the same states until time $T(n)$, for the tapes z and z'. Since B operate in time $T(n)$ and z is in T_2, it follows that z' is also accepted by B. This contradicts the fact that z' is not in T_2.]

Let $t(n)$ be the number of different sequences of states which c enters until time $T(n)$. Clearly, $t(n) \leq k^{T(n)}$. On the other hand (for any set S, let $|S|$ denote the number of elements of S.), $|W'(n)| = 2^{2^{2(n-1)}}$. Since $\lim_{n\to\infty} T(n)/2^{2n} = 0$ (by assumption of the lemma), it follows that $|W'(n)| > t(n)$ for lange n. Therefore, it follows that for large n there must exist two different tapes x, y in $W(n)$ such that

(i) both $x[(1,1,1),(2^{n-1},2^{n-1},2^{n-1})]$ and $y[(1,1,1),(2^{n-1},2^{n-1},2^{n-1})]$ and in $W'(n)$,

(ii) $x[(1,1,1),(2^{n-1},2^{n-1},2^{n-1})] \neq y[(1,1,1),(2^{n-1},2^{n-1},2^{n-1})]$, and

(iii) $(q_1(x), q_2(x), \ldots, q_{T(n)}(x)) = (q_1(y), q_2(y), \ldots, q_{T(n)}(y))$.

This contradicts the above Proposition 3.1, and thus the Part (2) of the lemma holds.

From Lemmas 3.1 and 3.2, we can get the following theorem.

Theorem 3.1. *Let $T(n)$ be a time function such that $\lim_{n\to\infty}[T(n)/2^{2n}] = 0$ and $T(n) \geq n (n \geq 1)$. Then £ [3-DFA] is imcomparable with £ [3-DUPCA$(T(n))$].*

Corollary 3.1. *£[3-DFA] is incomparable with £ [3-DUPCA(n)], which is the class of sets accepted by 3-DUPCA's operating in real time.*

Corollary 3.2. *£ [3-DFA] is incomparable with £[3-NUPCA(n)].*

4 Conclusion

In this paper, we investigated the accepting powers of bottom-up pyramid cellular acceptors with three-dimensional layers, and showed that the class of sets accepted by 3-*DFA*'s is incomparable with the class of sets accepted by 3-*DUPCA*'s which operate in time of order lower than the diameter of the input. It is still inknown whether the class of sets accepted by 3-*DUPCA*'s includes the class of sets accepted by 3-*DFA*'s.

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